A Hybrid Cellular Automaton Model of Smoldering Propagation in Forest Floor Duff

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Abstract

This study presents a general spatial model for the consumption of forest floor duff by smoldering combustion. Smoldering ground fires have an enormous impact upon the ecology and management of forest lands throughout the temperate zone. Here we propose a model to predict and better understand observed spatial patterns in duff consumption. The model avoids shortcomings often suffered by other models of duff consumption, such as site specificity, by using a small number of user-determined parameters (organic bulk density, moisture content, inorganic content, and duff depth) that can be estimated exclusively from field samples. A two-dimensional hybrid cellular automaton, coupling stochastic and deterministic processes, models the fuel bed. The model returns the stage of combustion, temperature, and moisture content across space and through time. Model output compares favorably to empirical and field studies concerning spatial aspects of duff consumption, predicting expected qualitative features. Modifications to the model are proposed that would advance understanding of the ecological impact of duff consumption.

Keywords: Duff consumption, smoldering combustion, organic soil, spatial heterogeneity, hybrid cellular automata, stochastic model.

Introduction

In recent years, the consumption of forest floor duff by smoldering combustion has come to be recognized as a vital component of the ecology and management of forest lands (Miyanishi 2001). In fire-excluded ecosystems, thick accumulations of duff have profoundly altered forest ecology in addition to contributing to high fuel loads (Miyanishi 2001; Varner et al. 2005; Hiers et al. 2007). Reducing these accumulations by prescribed burning, however, often presents serious difficulties since accumulated duff may smolder at the base of tree stems for hours leading to unacceptably high tree mortality (Ryan and Frandsen 1991; Swezy and Agee 1991; Varner et al. 2005, 2007). Additionally, smoldering duff may also cause secondary flaming combustion in other types of fuels leading to secondary fire fronts (Frandsen 1991).

Ignition of duff may occur either by direct contact with flaming litter or with organic debris that exhibit extended flaming combustion, particularly fallen branches and pine cones (Frandsen 1991; Fonda and Varner 2004). When ignition occurs, duff smolders down to the mineral soil leaving it exposed, resulting in a slow, outwardly propagating smoldering perimeter, exposing more mineral soil as it moves (Frandsen 1991). Extinction occurs when the smoldering front reaches conditions not suited to smoldering combustion: high moisture content and/or high inorganic content. If the heat generated by smoldering does not exceed the latent heat of vaporization required to drive the moisture from the duff, smoldering cannot continue (Frandsen 1991). Also, inorganic material does not burn and only absorbs heat energy so mineral content is also a limiting factor in the smoldering

process of forest duff (Frandsen 1991). The above processes reveal why moisture and mineral content are good predictors of the behavior of smoldering combustion in duff; two other predictors include the bulk density of organic material in the duff and duff thickness (Frandsen 1997; Miyanishi and Johnson 2002). These four duff characteristics are good predictors of the likelihood of consumption (Frandsen 1987, 1997; Miyanishi 2001; Miyanishi and Johnson 2002).

Smoldering combustion in materials such as polymer foams, sawdust, and tobacco has been studied both empirically and by explicitly modeling the combustion process (Bradbury et al. 1979; Ohlemiller 2002), but few, if any, address forest duff. Studies that specifically seek to understand duff consumption (Frandsen 1987, 1991, 1997, 1998; Miyanishi and Johnson 2002) often use peat as the material of study since peat has a high organic content and is structurally similar to forest duff (Frandsen 1987, 1991, 1998; Miyanishi 2001). Frandsen (1987, 1991, 1998) repeatedly notes that peat has similar structural characteristics to forest duff including particle size distributions. Miyanishi (2001) also cites studies showing that peat is chemically similar to the fermentation horizon in duff.

Here we develop a spatially explicit model for smoldering propagation through a fuel bed. Due to the complexity of combustion chemistry and the consequent difficulties in modeling combustion deterministically, propagation of the smoldering front is taken to be a stochastic process based upon conditions of the unconsumed duff. The ignition probability statement of Frandsen (1997) is used to determine the likelihood of propagation.

Since conditions ahead of the front change in time, particularly duff

moisture, it is necessary to model how the smoldering front imparts thermal energy to unconsumed duff. The model of Campbell et al. (1995), which is based upon the classic heat and moisture transport model of de Vries (1958), is a one-dimensional model of these processes in soils with high temperatures. The model developed here modifies the Campbell model to account for coupled heat and moisture transport in two dimensions.

Our objective was to construct a general model of forest floor duff consumption that is easy to use, modify, and incorporate into existing fire behavior models. To these ends, the model requires only eight parameters determined by the duff.

Methods

The fuel bed is modeled by a two-dimensional hybrid cellular automaton coupling stochastic and deterministic processes (Figure 1). A cellular automaton is a multi-dimensional lattice of cells that are updated in discrete time. The "hybrid" nomenclature emphasizes the interaction between deterministic and stochastic processes on the lattice. Hybrid cellular automata have been successfully applied to other important phenomena including tumor growth (Gerlee and Anderson 2007). To distinguish it from other models of smoldering propagation through a fuel bed, the model developed here will be referred to as the HCA model. In the HCA model, associated with each cell is a *state*. The state of each cell (representing a small patch of duff) is updated using information about itself and its *four nearest neighbors* (Figure 2).

The state of each cell is represented by a vector whose components are duff conditions that change in time: the stage of combustion; gravimetric moisture content (GMC); volumetric moisture content (VMC); and duff temperature. The combustion stage is updated stochastically (Figure 3) while the latter three are updated deterministically. Also associated with each cell are quantities that do not change in time: organic bulk density, mineral content, and duff thickness. Here, the lattice of cells is a square grid of uniformly spaced points in a plane with cell (1,1) acting as the origin. The vertical and horizontal distance between adjacent cells is denoted by Δx so that cell (i, j) occupies the point $(x_i, y_j) = ((i-1)\Delta x, (j-1)\Delta x)$. Each lattice point is interpreted as a small $\Delta x \times \Delta x$ area of duff (Figure 2). For a broad introduction to stochastic spatial models see Durrett (1999).

The combustion process in each cell is modeled by three stages: unburned, burning, and burned (Figure 3). Only cells undergoing combustion can ignite unconsumed cells. If one or more nearest neighbors of an unconsumed cell are burning, then the cell will undergo a transition from unburned to burning with a probability determined by the conditions of the cell. This probability is a maximum probability χ that is scaled back by the ignition probability statement given by Frandsen (1997) and depends upon duff conditions (Equation (1)). Thus the ignition probability of a cell having gravimetric moisture content γ, mineral content α, and organic bulk density ρ is assumed to be

$$\mathscr{P}(\text{Ignition}) = \frac{\chi}{1 + e^{-(B_0 + B_1\gamma + B_2\alpha + B_3\rho)}}. \tag{1}$$

The parameters B_0 B_1, B_2, B_3 are estimated using logistic regression on burn/no-burn data for a particular type of duff. The probability χ determines an uninhibited smolder velocity on the lattice. To calibrate the model to achieve a desired uninhibited smolder velocity (oven dry duff), the model is allowed to run multiple times with particular values for the ignition probability χ, spacing Δx, and update interval Δt. The smolder velocity is then taken to be the mean of a sufficient number of trials. With data for smolder velocity for varying values of the parameters χ, Δx, and Δt, the smolder velocity, Λ, is statistically modeled in terms of these parameters. Solving for χ in the resulting model yields an expression for the probability that gives the desired uninhibited smolder velocity:

$$\chi = \frac{\Delta t}{S_1 \Delta x} \left(\Lambda - S_0 - S_2 \Delta x - S_3 \Delta t \right),$$

where $S_0 = 0.0021$, $S_1 = 1.90$, $S_2 = -0.00476$, and $S_3 = -0.00185$.

Once a cell transitions from unburned to burning, the HCA model determines how long it burns. If a cell is burning, it will continue to burn provided there is fuel to be consumed and will pass to the burned state when no fuel remains. Instead of modeling directly the consumption of fuel, the time a cell undergoes combustion is assumed to be exponentially distributed with mean λ (Ross 2005). The mean smoldering time λ is assumed to be proportional to the amount of time it takes for a smoldering front to travel the distance Δx on the lattice which is on average $\frac{\Delta x}{\Lambda}$. This average smoldering time is then scaled by a dimensionless proportionality constant ϑ so that $\lambda = \vartheta \frac{\Delta x}{\Lambda}$.

Duff moisture is necessary for determining the ignition probability in unconsumed cells. Consequently, the HCA model monitors the amount of thermal energy imparted to the unconsumed duff by the smoldering front that then drives off moisture in neighboring cells. Therefore, the temperature T (Kelvin) of the duff for each time step is calculated. Duff thickness influences the efficiency at which the smoldering front imparts thermal energy to unconsumed duff. Thus also associated with each cell is a thickness τ (m). The smoldering boundary of thin duff is more prone to losing thermal energy to convective heat loss (Miyanishi and Johnson 2002), thermal energy that would otherwise be used to drive off moisture and sustain the combustion process. The HCA model accounts for this phenomenon by inhibiting the amount of thermal energy imparted to the soil by scaling the combustion temperature of the duff, T_c, by an *efficiency* factor that depends upon the depth associated with the ignited cell (described in detail later).

Temperature and Moisture Dynamics

The HCA model employs a continuous model of heat and moisture dynamics and the resulting system of partial differential equations is discretized. The soil (duff) is treated as a mixture of various components, namely mineral solids, organic solids, air, and water. The HCA model uses a modified version of the heat and moisture transport model in mineral soil developed by Campbell et al. (1994, 1995).

The HCA model modifies the model of Campbell et al. (1995) for a two-dimensional fuel bed of duff instead of a one-dimensional soil column. Temperature and moisture gradients drive both vertical and horizontal trans-

port of heat and moisture in porous media (de Vries 1958). Thus the one-dimensional vertical gradient $\frac{\partial}{\partial z}$ is replaced by the two-dimensional gradient operator $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$. Taking convective cooling at the surface of the duff and mineral soil into account, the continuous model for heat and moisture transport is

$$C\frac{\partial T}{\partial t} - Hd_w\frac{\partial \theta}{\partial t} = \nabla \cdot (K\nabla T) - \kappa_c C(T - T_a) \tag{2}$$

$$d_w\frac{\partial \theta}{\partial t} = -\nabla \cdot \left(\frac{V}{1 - \frac{p(T)}{P_a}}\nabla(p(T))\right) \tag{3}$$

for (x, y) in the unconsumed region of duff. The convective cooling constant, κ_c (hr^{-1}), is taken to be 0.1, and T_a is the ambient temperature (293.15 K). Notice that in the absence of space and moisture considerations, Equation (2) reduces to Newton's Law of Cooling with κ_c as the cooling constant. The burned region is updated stochastically. A continuous rule is not developed for how this region changes through time. The rule governing how the burned region changes through time is more naturally accomplished in a discrete setting. The temperature T is measured with the Kelvin (K) scale, θ is the volumetric moisture content of the soil ($\text{m}^3\ \text{m}^{-3}$), C is the volumetric heat capacity of the soil $\left(\text{J}\ \ \text{m}^{-3}\text{K}^{-1}\right)$, H is the latent heat of vaporization of water $\left(\text{J}\ \ \text{kg}^{-1}\right)$, d_w is the density of liquid water $\left(\text{kg}\ \ \text{m}^{-3}\right)$, p is the partial pressure of water vapor in the soil (Pa), P_a is ambient pressure (taken to be

standard pressure, 101,325 Pa), and V is the vapor conductivity of the soil $\left(\text{kg m}^{-1}\text{ Pa}^{-1}\text{ hr}^{-1}\right)$. Note that the HCA model uses different time units than Campbell et al. (1995) since smoldering combustion is a very slow process (linear smolder velocities are on the order of 10^{-2} m hr^{-1}). The term $-Hd_w\frac{\partial\theta}{\partial t}$ accounts for transport of thermal energy due to water vapor moving through the soil. The term $1/(1-\frac{p(T)}{P_a})$ is called the *Stefan correction* which is a mass flow correction that accounts for the flow of water vapor induced by the movement of air in the soil (Ghildyal and Tripathi 1987). Also, note that θ is the sum of the volumetric fractions of liquid water and precipitable water vapor (the volumetric fraction of condensed water vapor).

The thermal conductivity K (in J m $^{-1}$ hr^{-1}K^{-1}) is determined by soil type. Since both the chemical and physical structure of peat are similar to forest duff, the HCA model assumes that the physical characteristics of peat including mass and thermal transport properties are similar enough to act as a surrogate for forest duff. A temperature-dependent thermal conductivity model for soil (Campbell et al. 1994; Hiraiwa and Kasubuchi 2000; Tarnawski et al. 2000; Balland and Arp 2005) is necessary since Frandsen (1991) notes that smoldering ground fires elevate underlying mineral soil to temperatures above 300°C with temperatures as high as 600°C. The HCA model uses the temperature-dependent model of Campbell et al. (1994) to estimate the thermal conductivity of duff. The parameters for peat are used with the thermal conductivity model of Campbell et al. (1994).

The vapor conductivity V of the soil is expressed by Campbell et al. (1995) as $V = \frac{\xi\eta(\pi-\theta)M_wD_v}{RT}$ where M_w is the molecular weight of water $\left(\text{kg mol}^{-1}\right)$ and R is the universal gas constant $\left(\text{mol K}^{-1}\right)$. The tortuosity

correction is given by ξ and is taken to be 0.66 (Campbell et al. 1995). The tortuosity of a soil is the ratio of the straight line distance from one location to another to how far a particle actually travels (Hillel 1998). The vapor flow enhancement factor η is a tuning parameter that is taken to be 1 for most mineral soil applications (Campbell et al. 1995; Hillel 1998). However, since this effort concerns moisture transport in duff, a different value is used (see implementation). The expression $\pi - \theta$ is the air filled pore space of the soil. The diffusivity of water vapor in air D_v $\left(\mathrm{m}^{\ 2}\ \mathrm{hr}^{-1}\right)$ is given by $D_v = D_{v0}(\frac{P_0}{P_a})(\frac{T}{T_0})^{7/4}$ where D_{v0} is the diffusivity and P_0 is the ambient pressure at standard temperature and pressure.

An empirically derived equation used by Campbell et al. (1994) expresses the latent heat of vaporization in terms of temperature in the units of J mol^{-1} as $H = 45144 - 48(T - 273.15)$. The above is rescaled to J kg^{-1} by using the molecular weight of water.

Campbell et al. (1995) note that the relationship between p and θ is not unique. To account for this, the partial pressure of water is expressed as the product of relative humidity h and the saturation vapor pressure P (Pascals), $p = hP$. Campbell et al. (1995) note that the saturation vapor pressure of water is function of temperature alone and give an empirically derived formula for P in terms of T and is given by

$$P(T) = P_0 e^{13.3016S(T) - 2.042S(T)^2 + 0.26S(T)^3 + 2.69S(T)^4}$$

where $S(T) = 1 - \frac{373.15}{T}$. The humidity is expressed in terms of the temperature of the soil as $h = e^{\frac{M_w \psi}{RT}}$, where ψ is the water potential $\left(\mathrm{J}\ \mathrm{kg}^{-1}\right)$ of the

soil. The water potential for duff is allowed to vary between $\psi_{sat} = -10^{-2}$ J kg^{-1} (for saturated duff where saturation is defined to occur when the volumetric fraction of water equals the porosity of dry duff) and $\psi_0 = -10^6$ J kg^{-1} (for oven dry soil (Campbell et al. 1995)) according to the empirical relation $\psi(\theta) = a\theta^b$ for unsaturated soil where a and b are fitting parameters (Hillel 1998). The points (θ_{min}, ψ_0) and $(\theta_{sat}, \psi_{sat})$ on the estimated water retention curve determine the estimates for a and b (the field conditions of spruce/pine duff given by Frandsen (1997) induce the parameter estimates $a = -0.0062$, $b = -4.1024$). By using this relation, the HCA model assumes that moisture transfer takes place in unsaturated duff, and that water potential depends solely upon moisture content. This is reasonable since duff consumption takes place primarily in an unsaturated range of moisture (Hille and Stephens 2005).

Since moisture content is used to update ignition probability, it becomes unnecessary to update after a cell is undergoing consumption. In smoldering and consumed regions θ is set to a specified minimum value θ_{min} (taken to be 0.01). In the smoldering regions the temperature is set to the scaled temperature at which duff undergoes combustion,

$$(T_c - T_a)\left(\frac{1}{1 + \varepsilon_1 e^{-\varepsilon_2 \tau_t^{i,j}}}\right) + T_a,$$

where the unscaled combustion temperature T_c (for deep duff) is taken to be 400°C (Miyanishi 2001). The efficiency factor, $\frac{1}{1+\varepsilon_1 e^{-\varepsilon_2 \tau}}$, approaches 1 as depth increases, indicating a maximum efficiency of the smoldering front to impart thermal energy to the soil where ε_1 and ε_2 are efficiency

parameters (respectively taken to be 9 and 150 since a maximum efficiency of 1 is approached a depth of about 0.05 m). When extinction occurs, the temperature decays to the ambient temperature according to Newton's Law of Cooling. Temperature and VMC at the boundary of the domain are assumed to be the ambient temperature and initial VMC. The above establishes boundary conditions for the temperature and moisture dynamics at the ever-changing boundary of unconsumed duff.

Equations (2) and (3) and the boundary conditions described above serve as the continuous thermal and moisture transport model in unconsumed duff. The model equations are discretized to develop update rules for duff temperature and moisture on the cellular lattice described above resulting in the HCA model. The discretization is accomplished by using a forward difference in time and a centered difference in space.

Implementation

The HCA model was implemented in MATLAB using Monte Carlo methods to simulate ignition probabilities determined by duff conditions. To improve the run time and stability in the numerical scheme for solving equations (2) and (3), several restrictions were imposed. The non-Laplacian terms were ignored when expanding the right hand side of [2] into its component derivatives. This change had little, if any, perceptible effect upon the behavior of the model. The thermal conductivity was restricted to values between 900 (thermal conductivity of organic material) and 18,000 J m $^{-1}$ hr^{-1}K^{-1} (upper bound used by Campbell et al. (1995)). The volumetric moisture content was also restricted to values no lower than θ_{min}. Since the model of Campbell

et al. (1995) predicts a buildup of moisture ahead of a heat pulse, followed by decay, an additional restriction was made to eliminate this buildup; the updated volumetric moisture content was not allowed to exceed the volumetric moisture content of the previous time step so that moisture is a decreasing function of time. Another restriction made was to let the ambient temperature act as a lower bound when updating temperature. With a vapor flow enhancement factor of 1, the unrestricted temperature calculations predicted freezing temperatures in some cells (the numerical scheme still appeared to be stable). However, by using a *reduced* vapor flow enhancement factor of 1/3, this phenomenon was greatly reduced, with the most extreme cooling being only fractions of a degree below ambient temperature. Since duff is composed mostly of organic solids which serve to increase water retention (Koorevaar et al. 1983), this reduction is not unreasonable. Therefore, the model assumes a reduced vapor flow enhancement factor of 1/3 in addition to a setting a lower bound on temperature. Imposing this restriction affords the user more flexibility in choosing a vapor flow enhancement factor since the model behaves well even when resetting the vapor flow enhancement factor back to 1.

To compare the HCA model to empirical and laboratory studies, uniform initial conditions were used since burn studies often use peat samples of uniform composition (Frandsen 1987, 1991, 1998; Miyanishi 2001). For all simulations, duff conditions were initialized to the field conditions for bulk density and inorganic content for spruce/pine (*Picea/Pinus*) duff reported by Frandsen (1997). Unless otherwise noted, each model run used

the regression coefficients for spruce/pine duff reported by Frandsen (1997).

Results

The coupled combustion state and heat and moisture dynamics (Figure 4) predict that regions exposed to the smoldering front are significantly drier than unexposed regions, that is, a drying region ahead of the smoldering front.

The end stage of the smoldering process (when the process is allowed to proceed until extinction) is seen in Figures 5 and 6. Four consecutive simulations were run for moist duff conditions (105% GMC) using both initial burn configurations. Each simulation was allowed to proceed until extinction occured.

The amount of duff consumed varies with the moisture and inorganic ratio (the ratio of the mass of water and inorganic solids to organic mass). The model was repeatedly run incrementing these parameters. For each run the initial simulated duff conditions were set to be uniform as in Frandsen (1987) and simulated bulk density for each run was the average field bulk density (116 kg m^{-3}) for spruce/pine duff reported by Frandsen (1997). The model was run for moisture ratios from 0 to 2 and inorganic ratios from 0 to 6, each in increments of 0.1 and the average proportion of duff consumed over fifty trials was recorded.

The simulated smolder velocity was measured using the same method described in the model formulation, with the exception that the dynamic duff conditions also determined the spread probability. Fifty trials were

conducted and then averaged to produce a predicted simulated smolder velocity at each combination of inorganic and moisture ratio (Figure 8).

To test whether or not depth has any effect on duff consumption, the model was run for varying depth and volumetric moisture content and the average proportion of duff consumed over fifty trials was recorded for each combination of these. The results clearly demonstrate that depth does affect the behavior of the model, with increasing depth resulting in sustained smoldering propagation at higher moisture contents (Figure 9).

Discussion

In order to accurately model the spatial patterns observed in a smoldering ground fire, the phenomenon of a drying front in duff is a key aspect this model seeks to capture. The simulated temperature distribution (Figure 4) displays isolated patches of smoldering combustion indicating qualitative agreement with thermal images of actual smoldering ground fires. In the field, unburned "islands" of duff are commonly observed (Miyanishi and Johnson 2002; Knapp and Keeley 2006); the HCA model predicts this phenomenon near the limits of smoldering combustion (Figure 6).

The results of the simulations predicting limits of smoldering combustion (Figure 7) are qualitatively similar to empirical results found in Frandsen (1987) in which peat samples with known uniform composition were exposed to an ignition source and a result of burn/no burn was recorded. Frandsen observed that the limits of smoldering combustion are described well by a line of negative slope with positive intercepts. The HCA model predicts a similar

response in spruce/pine duff where the limits of smoldering combustion are similarly described (Figure 7).

It is apparent that there is overall qualitative agreement between the simulated smolder velocities and the smolder velocity predicted by the statistical model of Frandsen (1991). However, the delayed effect of the inorganic ratio accounted for in Frandsen's model is not present in the HCA model; Frandsen's predicted smolder velocity is not affected by the inorganic ratio until it reaches a value of 1. Also, the simulated smolder velocity does not appear to decrease until close to the limits of smoldering combustion where it then falls off rather sharply, whereas Frandsen's model predicts a linearly decreasing smolder velocity up to and past the limits of smoldering combustion. Also, Frandsen's model predicts a maximum smolder velocity of about 0.025 m hr^{-1}, whereas, the HCA model simulates a slightly larger maximum smolder velocity of about 0.028 m hr^{-1}.

Although the model is two-dimensional, it accounts for the effect of depth (third-dimension) as well by means of the efficiency function describing efficiency versus depth, described in the methods section. The smoldering perimeter more effectively transmits heat to the unconsumed duff due to less heat being carried off by convection (Miyanishi and Johnson 2002). The results suggest that increasing duff depth allows for sustained combustion at higher moisture contents which is in line with empirical studies (Miyanishi and Johnson 2002).

With increasing depth, the efficiency with which the smoldering front imparts thermal energy to the unconsumed duff also increases (with a limiting value of 1) (Figure 9). Since the relationship between depth and efficiency

is assumed to be a strictly increasing relationship, depth indirectly accounts for efficiency. Hence, at an efficiency (depth) near zero, the temperature has very little effect with little moisture being driven off ahead of the front. As the maximum efficiency is approached (deeper duff), however, the temperature takes effect and drives off more moisture ahead of the front so that more duff is consumed. Thus, temperature and moisture dynamics are worthy of consideration in any spatially explicit model of duff consumption.

Modeling the fuel bed as a lattice of cells offers multiple advantages. Stochastic and deterministic models are easily coupled and integrated into the model. Also, improved models (along with their numerical implementation) for the various phenomena considered (particularly heat and moisture transport) are easily integrated into the model. Additionally, modifications are easily made to the model to answer other important questions concerning duff consumption.

By considering stand characteristics such as tree stem density, stem and crown diameters, and how duff depth and moisture vary in relation to these quantities (Hille and Stephens 2005), an L by L meter area of forest may be simulated by stochastically initializing both of the above quantities jointly with duff characteristics. Once a rule is established for how contact with smoldering duff influences whether or not a tree dies (by duration of and area of exposure, bark thickness, temperature of smoldering, etc.), a ground fire may be simulated and an estimate of tree mortality caused by exposure to smoldering duff may be generated by repeated simulation.

Organic bulk density, mineral content, moisture content and thickness of the duff layer may be stochastically initialized for each cell to simulate a

typical fuel bed found in the field. The model encounters instabilities with large variation in organic bulk density,a shortcoming that may be remedied by neglecting the volumetric fraction of air in the duff when calculating volumetric heat capacity.

The two-dimensional lattice of cells can also model a vertical cross section of duff rather than a horizontal layer as presented here. With minor modifications taking orientation and the heterogeneity of the various organic soil horizons into account, repeated simulation could predict duff depth reduction under a known set of conditions. Unlike the model representing a horizontal duff layer, the influence of the distinct structural differences between the duff horizons upon heat and mass transport could be explicitly accounted for, particularly differences in quantities such as bulk density (affecting the air-filled pore space (Ochsner et al. 2001)) that influence mass and thermal transport. Repeated simulation of the model would not only offer mean and variance estimates of duff depth reduction, but would also predict a distribution for duff depth reduction. A one-dimensional lattice model of a soil column might also be considered. Duff depth reduction could also be estimated by introducing a slight modification to the model in which the depth of each cell decreases in a probabilistic fashion (Holt 2008).

The HCA model could also be useful in predicting the effects of prescribed burning on post-burn spatial heterogeneity. Fuel continuity caused by fire exclusion and the consequent lack of spatial heterogeneity in fuels and vegetation is favorable to some organisms, particularly those that flourish in duff. However, tree and herbaceous species that do not establish well on duff are in some cases virtually unable to propagate (Miyanishi 2001)

which illustrates how a lack of spatial heterogeneity can lead to decline in biological diversity (Knapp and Keeley 2006). Thus spatial heterogeneity, as measured by the variability and "patchiness" in ground cover, canopy cover, and other structural characteristics, is an important index of forest health. Low fuel loads believed to exist before fire exclusion are believed to have resulted in greater spatial heterogeneity in vegetation and fuels (Knapp and Keeley 2006). Knapp and Keeley (2006) showed that it is possible to approximate the degree of spatial heterogeneity believed to exist before fire exclusion suggesting that computer models would be useful in predicting the degree of post-burn heterogeneity. Patch size distributions generated by the model developed here offer measures of spatial heterogeneity in ground fuels. The HCA model in conjunction with pair approximation techniques reviewed by Sato and Iwasa (2000) used to analyze spatial heterogeneity in lattice models may also prove to be a powerful combination in predicting and characterizing spatial heterogeneity in ground fuels.

Conclusions

The HCA model of smoldering propagation in forest floor duff is easy to use, non-site-specific, and spatially and temporally accurate. The model predicts expected qualitative features concerning spatial patterns of duff consumption. The qualitative agreement with laboratory and field studies affirms that spatial patterns predicted by the model are indeed valid. A particularly appealing aspect of the model is that it is versatile. It is a general model from which a site-specific model may be developed by considering a

small number of easily measured, but key, parameters. The versatility of the model is not limited to its generality; the model is easily modified to account for other phenomena important to management and ecology, particularly, tree mortality, duff depth reduction, and post-burn spatial heterogeneity. These potential models, when incorporated into models concerning other aspects of prescribed and wildfire, could help foresters and ecologists better understand and simulate a process that has profound implications for forest health.

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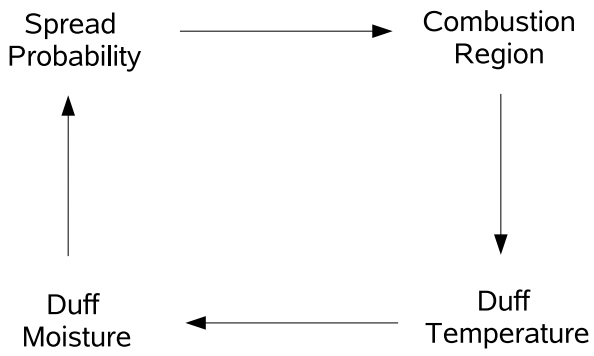

Figure 1: Overall model structure for predicting the spatial pattern of forest floor duff consumption.

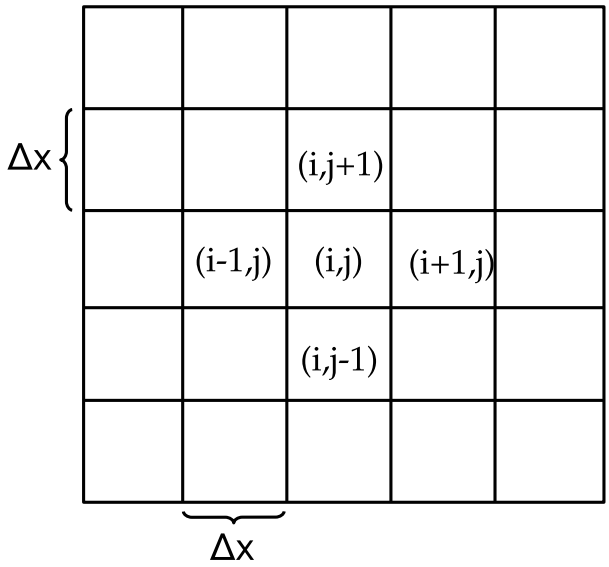

Figure 2: Cell (i, j) and its four nearest neighbors representing the cellular automaton approach used here to model smoldering propagation in forest floor duff.

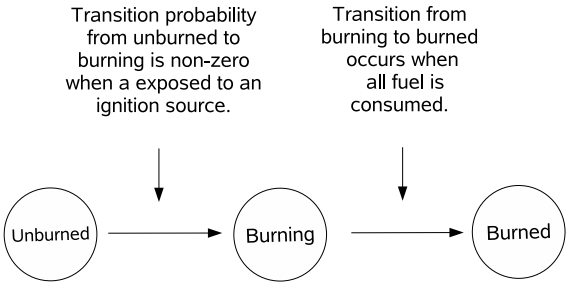

Figure 3: Three stage model of the combustion process in forest floor duff.

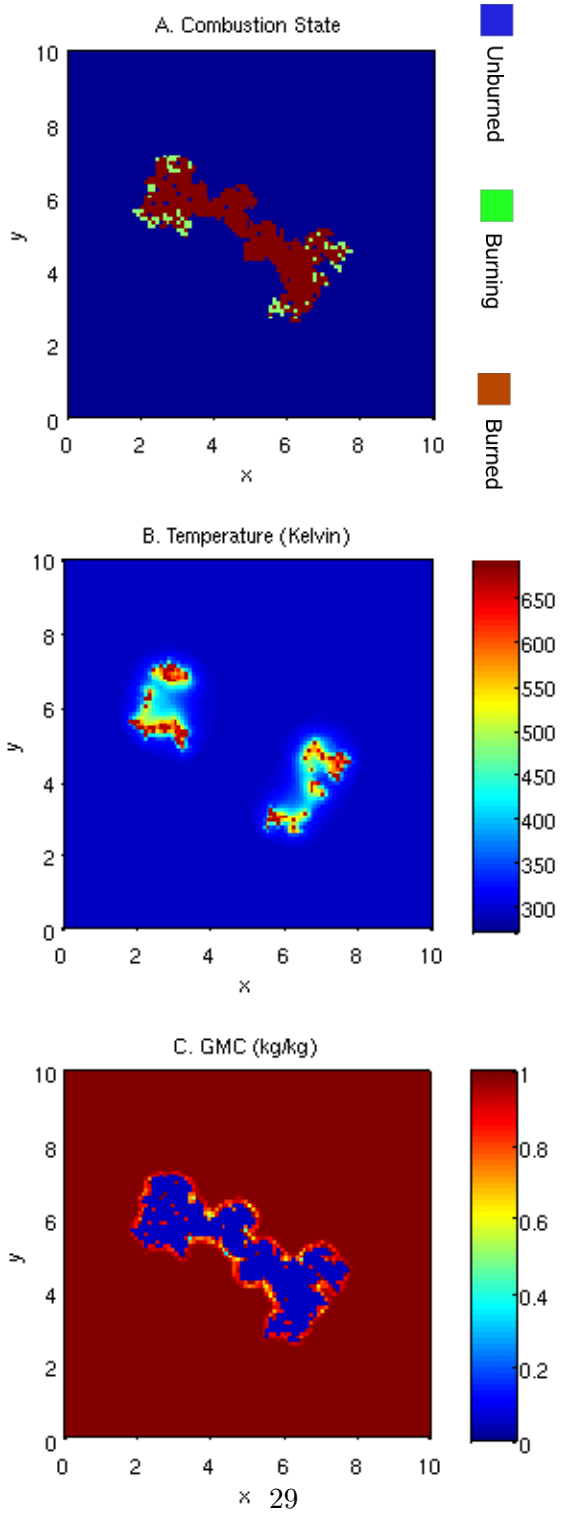

Figure 4: Simulated combustion, heat, and moisture dynamics of forest floor duff using a hybrid cellular automaton model. The predicted drying front is seen in panel C displaying moisture content across space. $\gamma = 1$ kg kg^{-1}, $\rho = 116$ kg m^{-3}, $\alpha = 0.307$ kg kg^{-1}, and $\tau = 0.15$ m.

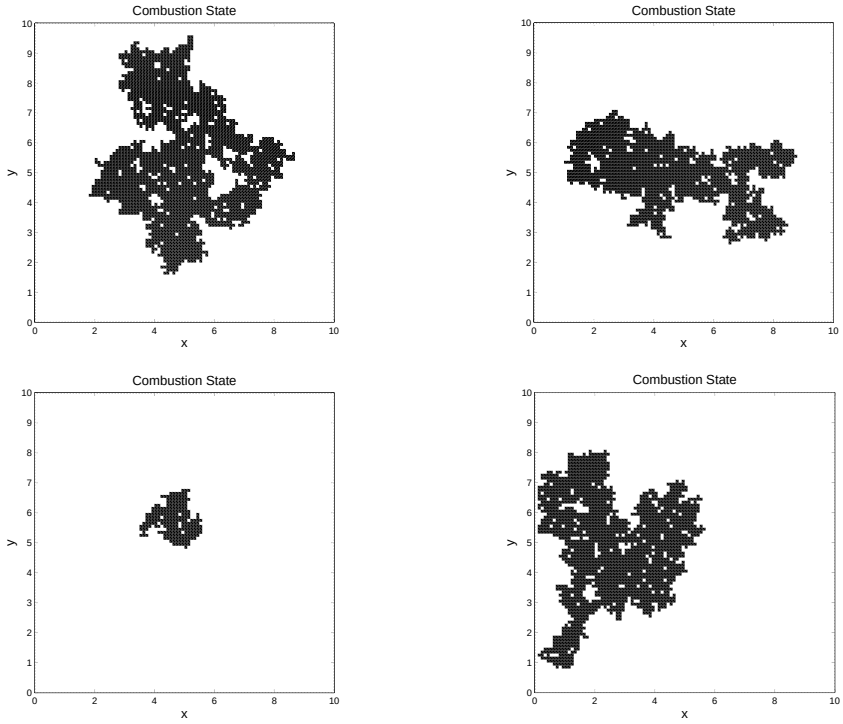

Figure 5: The resulting burn pattern for four consecutive model runs with very moist duff allowing the simulation to proceed until extinction. Ignition was initiated in the center for each run. The initial duff conditions were $\gamma = 1.05$ kg kg^{-1}, $\rho = 116$ kg m^{-3}, $\alpha = 0.307$ kg kg^{-1}, and $\tau = 0.15$ m.

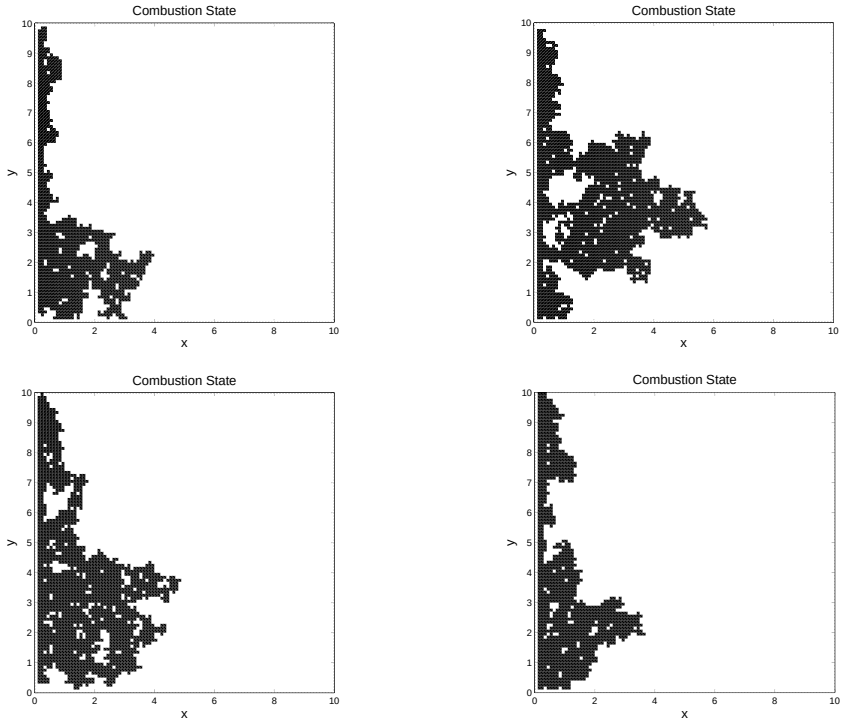

Figure 6: The resulting burn pattern for four consecutive model runs with very moist duff allowing the simulation to proceed until extinction. Ignition was initiated at the left edge for each run. The initial duff conditions were $\gamma = 1.05$ kg kg^{-1}, $\rho = 116$ kg m^{-3}, $\alpha = 0.307$ kg kg^{-1}, and $\tau = 0.15$ m.

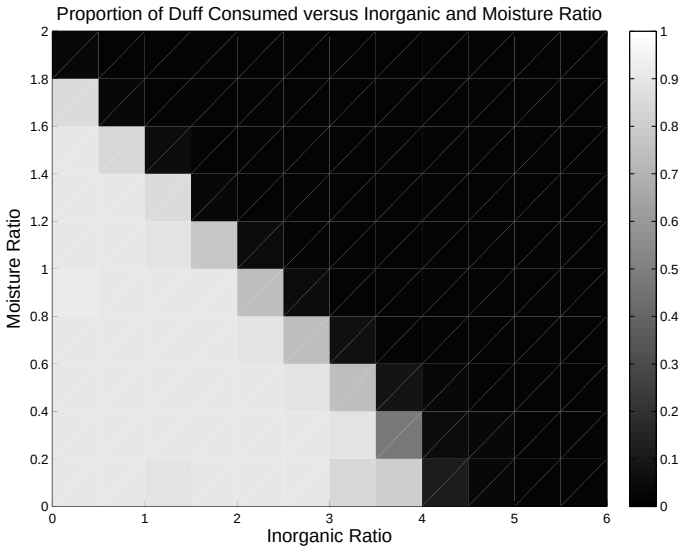

Figure 7: The proportion of a simulated 10m by 10m patch of spruce/pine duff consumed by smoldering combustion expressed in terms of inorganic and moisture ratio. Each point represents the average of fifty trials. $\rho = 116$ kg m^{-3}, $\tau = 0.15$ m.

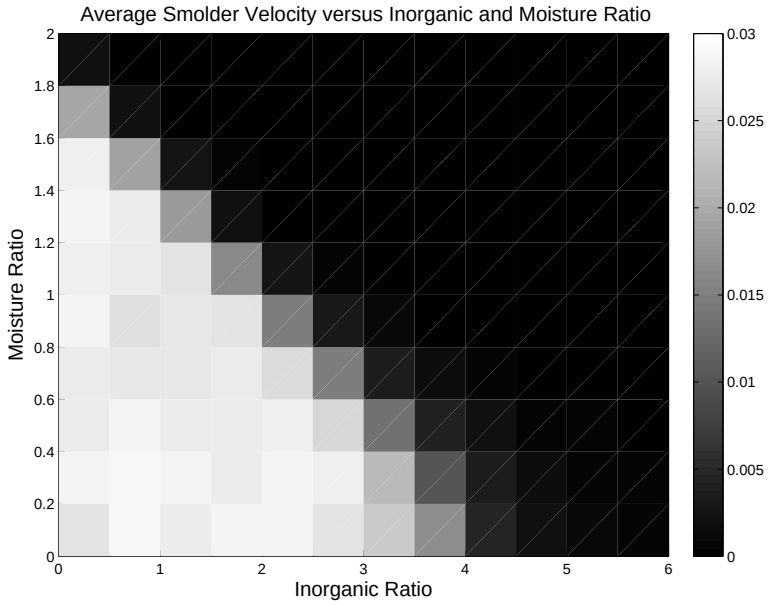

Figure 8: Simulated smolder velocities for spruce/pine duff, $\rho = 116$ kg m^{-3}, $\tau = 0.15$ m. Each point represents the average of fifty trials.

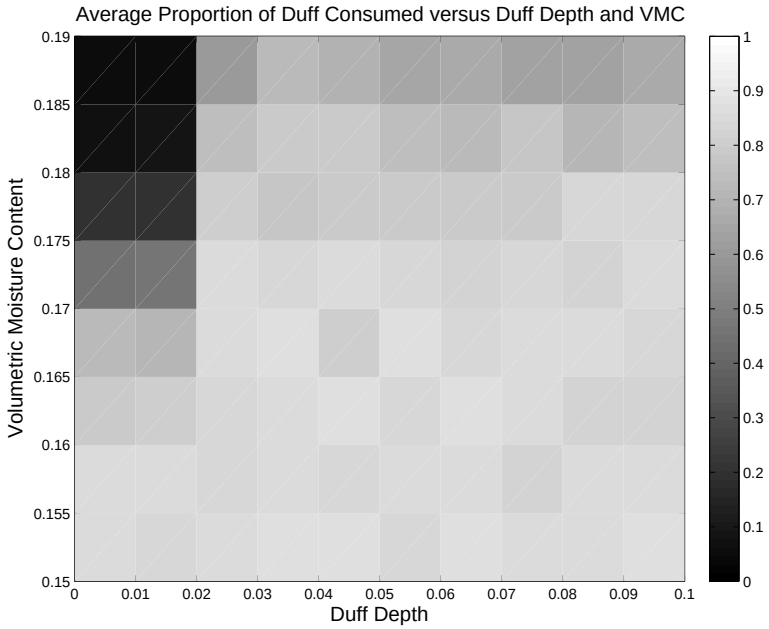

Figure 9: The average proportion of a simulated 1m by 1m patch of spruce/pine duff consumed completely by smoldering combustion expressed in terms of initial duff depth and volumetric moisture content. Ignition was initiated at the left edge. Each point represents the average of fifty trials. $\rho = 116$ kg/m^3, $\alpha = 0.307$ kg/kg.