POCKET CHANGE

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Introduction:

At the beginning of the semester, my group set out to find out what the most common denomination of coin currency is in everyday circulation and what dates are most common in each denomination. We also planned to compare the most common dates of each different denomination and determine why these dates are the most common. The kinds of coins that we will be analyzing are United States Pennies, Nickels, Dimes, and Quarters. To accomplish this, we collected a large sample of coins from daily transactions, selected a random sample from our large sample, and recorded their denomination and year minted. Then we graphed, tested, and analyzed the data. Before beginning our study, we hypothesized that the majority of coins that we found would be from the last 10 or so years because the more recent years haven't had the same opportunity to get taken out of circulation as older coins have.

Definitions and Assumptions:

Our defined population for this study is the total amount of United States coins in circulation. This includes pennies, nickels, dime, and quarters. We excluded half dollars and dollar coins because of their lack of popularity and circulation. They may have skewed our data if they were included.

In order to run tests on our samples, we had to assume that they were an accurate representation of the total population of coins. However, it would be extremely difficult to estimate the quantities of all of the coins in circulation without collecting an extremely large portion of the total population of coins. The sampling that my group conducted in this experiment cannot be considered as a true SRS of the total population of coins in the united states. There are many reason why collecting the coins from only one county might not accurately represent all of the coins in the country. It is possible that some coins circulate faster than others because they are more convenient amounts for typical transactions. This means that the sample that we will be collecting could be biased because it over-represents the coins that circulate faster. Perhaps in this part of the country, more of a particular coin is used, this would also create a bias. Therefore, it would be safe to say that our sample is not an SRS. But, for the sake of the experiment, we are going to assume that our data is accurate and we will use it just as we would use a real SRS.

When using an Analysis of Variance (ANOVA) test, it is assumed that the distributions of samples are normal. Thankfully, because my data is not normal, the F-statistic is relatively robust and resistant to violations of normality. Because I have collected fairly large and mostly equal sample sizes, and each of the sample distributions are similar in shape, I can use the ANOVA test and be confident that my F-statistic is fairly accurate. The ANOVA test is not very sensitive to deviations from normality.

Sampling Design and Methodology:

Throughout the semester, our individual group members, a few friends, and some family members had collected and saved all of the change that we acquired from making everyday transactions. This includes the change from transactions at places like grocery stores, clothing stores, gas stations, and restaurants. Each coin has information like the year it was minted and the location where it was minted. The denomination of each coin, the year it was minted, and mint-mark was documented and analyzed. Once we acquired a large portion of coins, we assigned a number to each coin and randomly selected a large sample using a list randomizer from random.org. We then recorded the total amount of coins, the total amount of each denomination of coin, and the amount of these coins that are a certain mintmark and year in our random sample. This information about the data was documented on a spreadsheet.

Analysis:

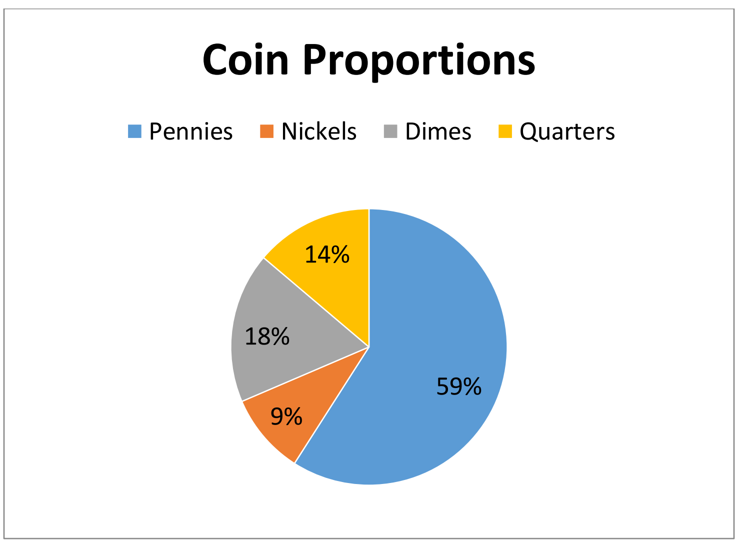
1. In total, our random sample ended up being 1600 coins. To begin our analysis of these coins we decided to try to determine what the proportion of our sample was pennies, nickels, dimes, and quarters. After counting the coins we found that we had 945 pennies, 152 nickels, 282 dimes, and 221 quarters. That is almost exactly $100 in coins! ($100.50). To determine the sample proportions ($\hat{p}$) we used the equation: $\hat{p}$ = number of successes / sample size.

Pennies: 945 / 1600 = 0.59

Nickels: 152 / 1600 = 0.095

Dimes: 282 / 1600 = 0.176

Quarters: 221 / 1600 = 0.138



2. Just out of curiosity, we decided to try to calculate what the true proportion of each denomination of coin is using our sample proportions. To find a 95% confidence interval for the true proportion we used the equation: $\hat{p} \pm z^* \sqrt{\hat{p}(1 - \hat{p}) / n}$.

Pennies: $0.59 \pm 1.96 \sqrt{0.59(1 - 0.59) / 945} = 0.59 \pm 1.96 (0.015999) = 0.59 \pm 0.0314 = (.559, .6214)$

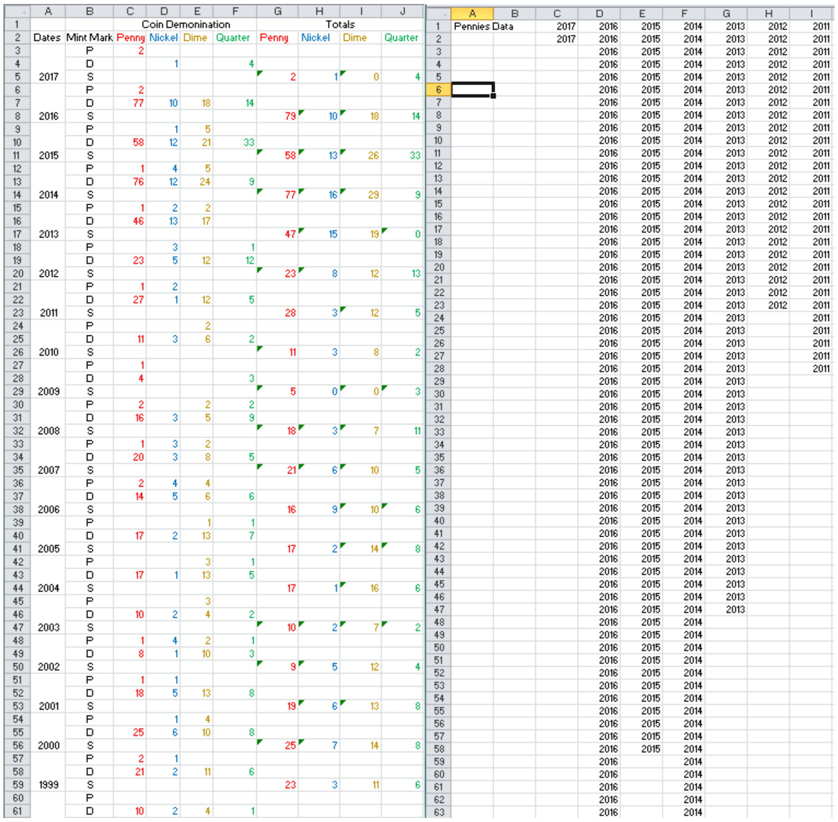
Nickels: $0.095 \pm 1.96 \sqrt{0.095(1 - 0.095) / 152} = 0.095 \pm 1.96 (0.0238) = 0.095 \pm 0.0466 = (.0484, .142)$

Dimes: $0.176 \pm 1.96 \sqrt{0.176 (1 - 0.176) / 282} = 0.176 \pm 1.96 (0.0227) = 0.59 \pm 0.0444 = (.132, .22)$

Quarters: $0.138 \pm 1.96 \sqrt{0.138 (1 - 0.138) / 221} = 0.138 \pm 1.96 (0.0232) = 0.59 \pm 0.0455 = (.0925, .184)$

Therefore, the average margin of error for each of these estimations is around 4%. If we had collected just a couple hundred more coins, we could have brought it down to a 3% margin of error. Therefore, we can be 95% certain that our sample proportions predict the true population proportion with a margin of 4%.

3. The second part of our experiment was to find what the average dates were for each of the denominations in our sample and how the mean dates compared with each other. Therefore, we documented the dates of each of the 1600 coins that we collected. This was a long and messy process. Many hours were spent writing down a coin's year and transferring it to a spreadsheet.



	A	B	C	D	E	F	G	H	I	J
1			Coin Demonination				Totals			
2	Dates	Mint Mark	Penny	Nickel	Dime	Quarter	Penny	Nickel	Dime	Quarter
3		P	2							
4		D		1		4				
5	2017	S					2	1	0	4
6		P	2							
7		D	77	10	18	14				
8	2016	S					79	10	18	14
9		P		1	5					
10		D	58	12	21	33				
11	2015	S					58	13	26	33
12		P	1	4	5					
13		D	76	12	24	9				
14	2014	S					77	16	29	9
15		P	1	2	2					
16		D	46	13	17					
17	2013	S					47	15	19	0
18		P		3		1				
19		D	23	5	12	12				
20	2012	S					23	8	12	13
21		P	1	2						
22		D	27	1	12	5				
23	2011	S					28	3	12	5
24		P			2					
25		D	11	3	6	2				
26	2010	S					11	3	8	2
27		P	1							
28		D	4			3				
29	2009	S					5	0	0	3
30		P	2		2	2				
31		D	16	3	5	9				
32	2008	S					18	3	7	11
33		P	1	3	2					
34		D	20	3	8	5				
35	2007	S					21	6	10	5
36		P	2	4	4					
37		D	14	5	6	6				
38	2006	S					16	9	10	6
39		P			1	1				
40		D	17	2	13	7				
41	2005	S					17	2	14	8
42		P			3	1				
43		D	17	1	13	5				
44	2004	S					17	1	16	6
45		P			3					
46		D	10	2	4	2				
47	2003	S					10	2	7	2
48		P	1	4	2	1				
49		D	8	1	10	3				
50	2002	S					9	5	12	4
51		P	1	1						
52		D	18	5	13	8				
53	2001	S					19	6	13	8
54		P		1	4					
55		D	25	6	10	8				
56	2000	S					25	7	14	8
57		P	2	1						
58		D	21	2	11	6				
59	1999	S					23	3	11	6
60		P								
61		D	10	2	4	1				

	A	B	C	D	E	F	G	H	I
1	Pennies Data		2017	2016	2015	2014	2013	2012	2011
2			2017	2016	2015	2014	2013	2012	2011
3				2016	2015	2014	2013	2012	2011
4				2016	2015	2014	2013	2012	2011
5				2016	2015	2014	2013	2012	2011
6				2016	2015	2014	2013	2012	2011
7				2016	2015	2014	2013	2012	2011
8				2016	2015	2014	2013	2012	2011
9				2016	2015	2014	2013	2012	2011
10				2016	2015	2014	2013	2012	2011
11				2016	2015	2014	2013	2012	2011
12				2016	2015	2014	2013	2012	2011
13				2016	2015	2014	2013	2012	2011
14				2016	2015	2014	2013	2012	2011
15				2016	2015	2014	2013	2012	2011
16				2016	2015	2014	2013	2012	2011
17				2016	2015	2014	2013	2012	2011
18				2016	2015	2014	2013	2012	2011
19				2016	2015	2014	2013	2012	2011
20				2016	2015	2014	2013	2012	2011
21				2016	2015	2014	2013	2012	2011
22				2016	2015	2014	2013	2012	2011
23				2016	2015	2014	2013	2012	2011
24				2016	2015	2014	2013		2011
25				2016	2015	2014	2013		2011
26				2016	2015	2014	2013		2011
27				2016	2015	2014	2013		2011
28				2016	2015	2014	2013		2011
29				2016	2015	2014	2013		
30				2016	2015	2014	2013		
31				2016	2015	2014	2013		
32				2016	2015	2014	2013		
33				2016	2015	2014	2013		
34				2016	2015	2014	2013		
35				2016	2015	2014	2013		
36				2016	2015	2014	2013		
37				2016	2015	2014	2013		
38				2016	2015	2014	2013		
39				2016	2015	2014	2013		
40				2016	2015	2014	2013		
41				2016	2015	2014	2013		
42				2016	2015	2014	2013		
43				2016	2015	2014	2013		
44				2016	2015	2014	2013		
45				2016	2015	2014	2013		
46				2016	2015	2014	2013		
47				2016	2015	2014	2013		
48				2016	2015	2014			
49				2016	2015	2014			
50				2016	2015	2014			
51				2016	2015	2014			
52				2016	2015	2014			
53				2016	2015	2014			
54				2016	2015	2014			
55				2016	2015	2014			
56				2016	2015	2014			
57				2016	2015	2014			
58				2016	2015	2014			
59				2016		2014			
60				2016		2014			
61				2016		2014			
62				2016		2014			
63				2016		2014			

Once all of the dates were collected physically and electronically, we inserted it into the "**Descriptive One Variable Statistics Calculator**" found at holt.blue. This gave us an x̄, s, n, five number summary, and a histogram for each of our denominations.

4. Pennies:

Number of Data Points: n = 945

Mean: x̄ = 1998.352380952381

Standard Deviation: s = 14.770549357735383

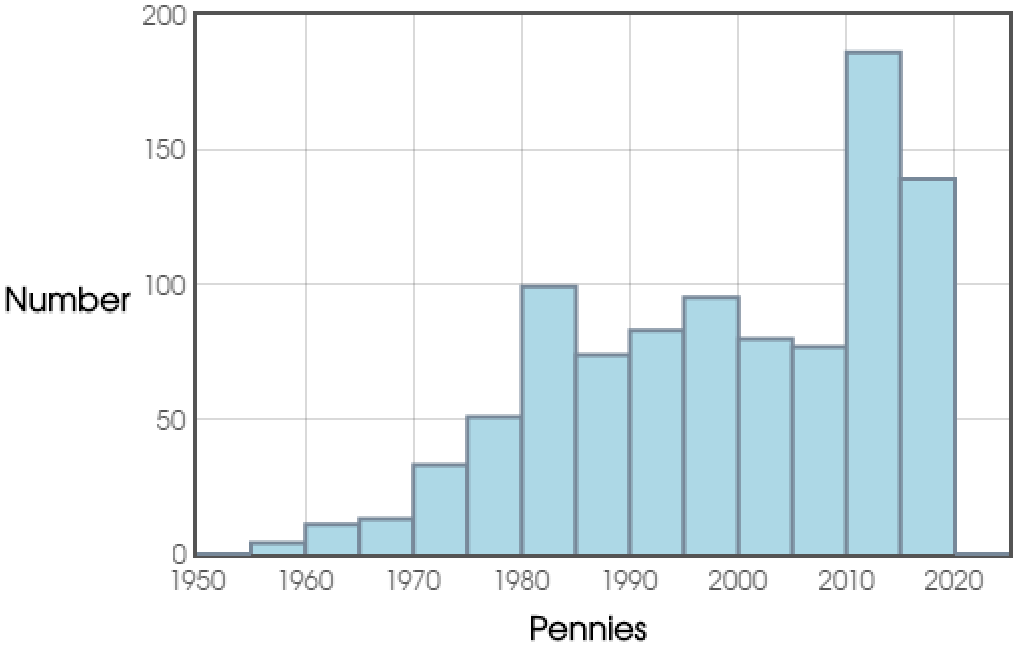
Minimum: Min = 1956

1st Quartile: Q1 = 1986

Median: M = 2000

3rd Quartile: Q3 = 2013

Maximum: Max = 2017



5. Nickels

Number of Data Points: n = 152

Mean: x̄ = 2003.171052631579

Standard Deviation: s = 13.08139385194844

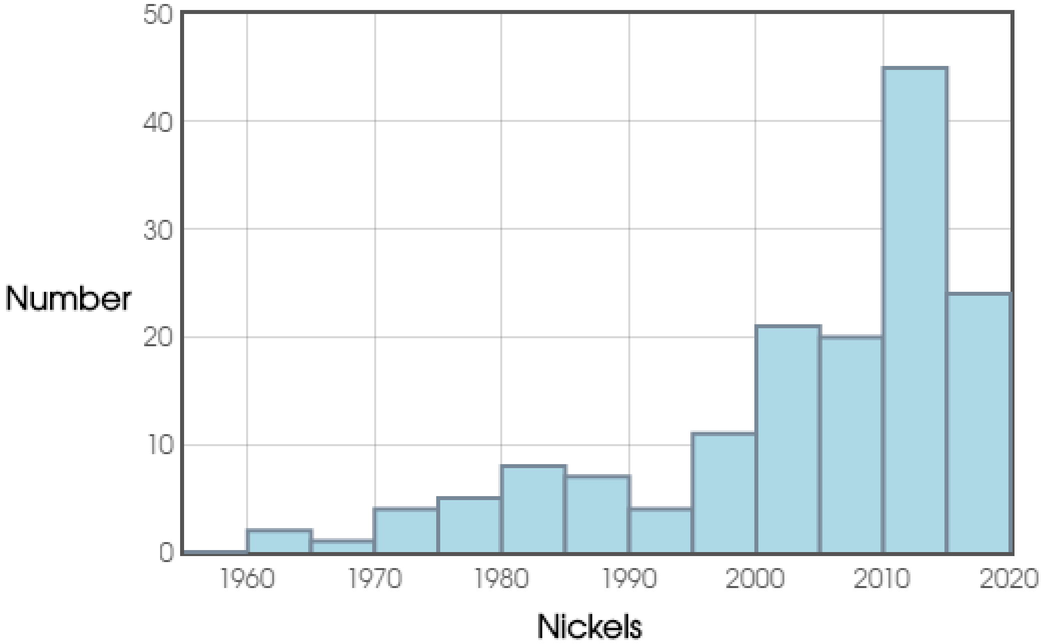
Minimum: Min = 1963

1st Quartile: Q1 = 1998

Median: M = 2007

3rd Quartile: Q3 = 2014

Maximum: Max = 2017



6. Dimes

Number of Data Points: n = 282

Mean: x̄ = 2005.1950354609928

Standard Deviation: s = 10.419710491406024

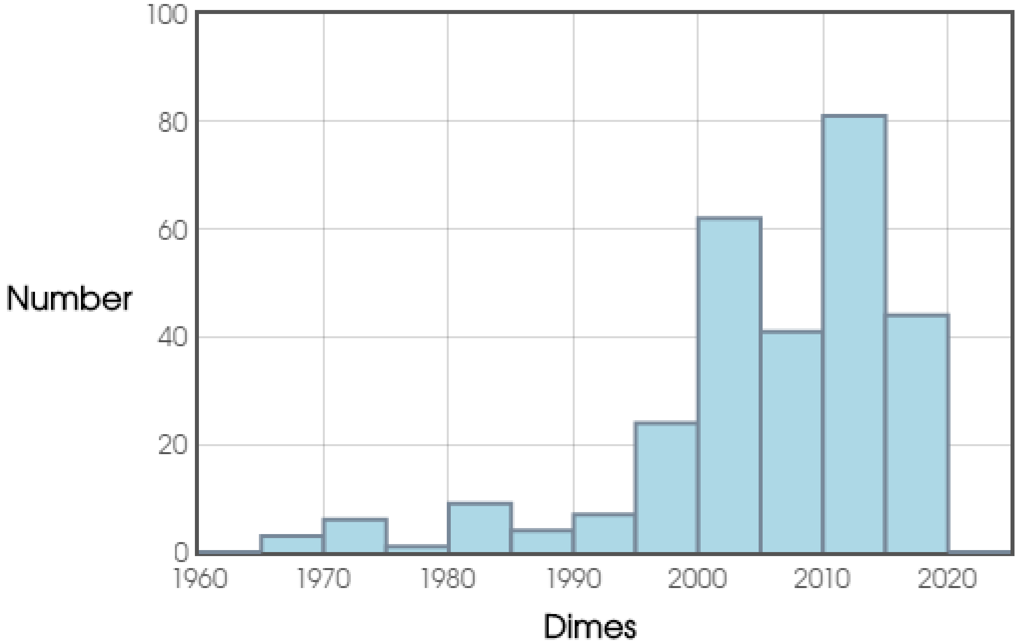
Minimum: Min = 1965

1st Quartile: Q1 = 2001

Median: M = 2007

3rd Quartile: Q3 = 2014

Maximum: Max = 2016



7. Quarters:

Number of Data Points: n = 221

Mean: $\bar{x}$ = 2000.9909502262444

Standard Deviation: s = 14.100287215071608

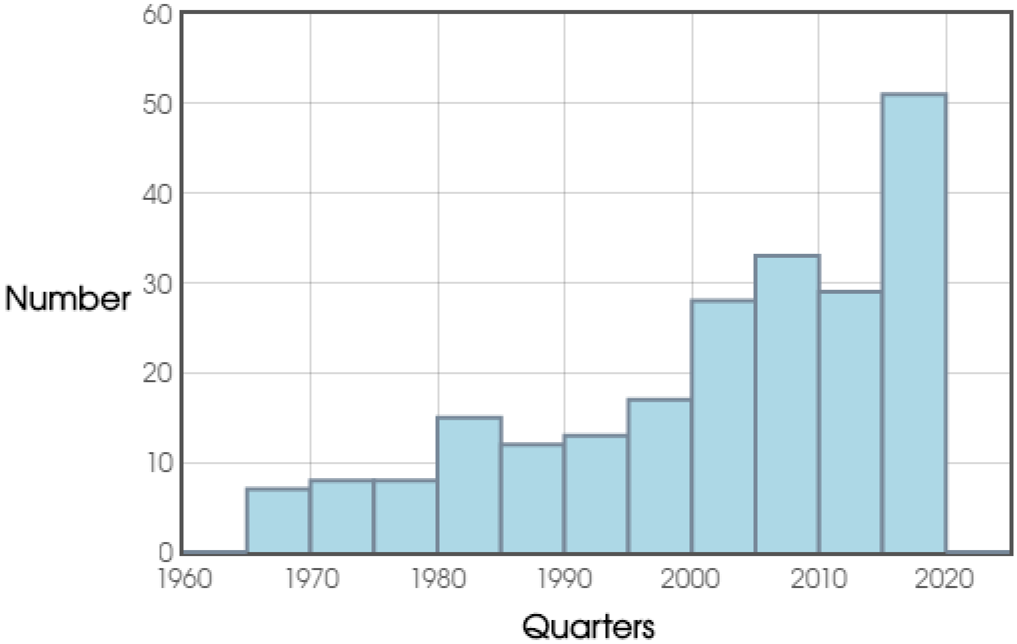
Minimum: Min = 1965

1st Quartile: Q1 = 1992.5

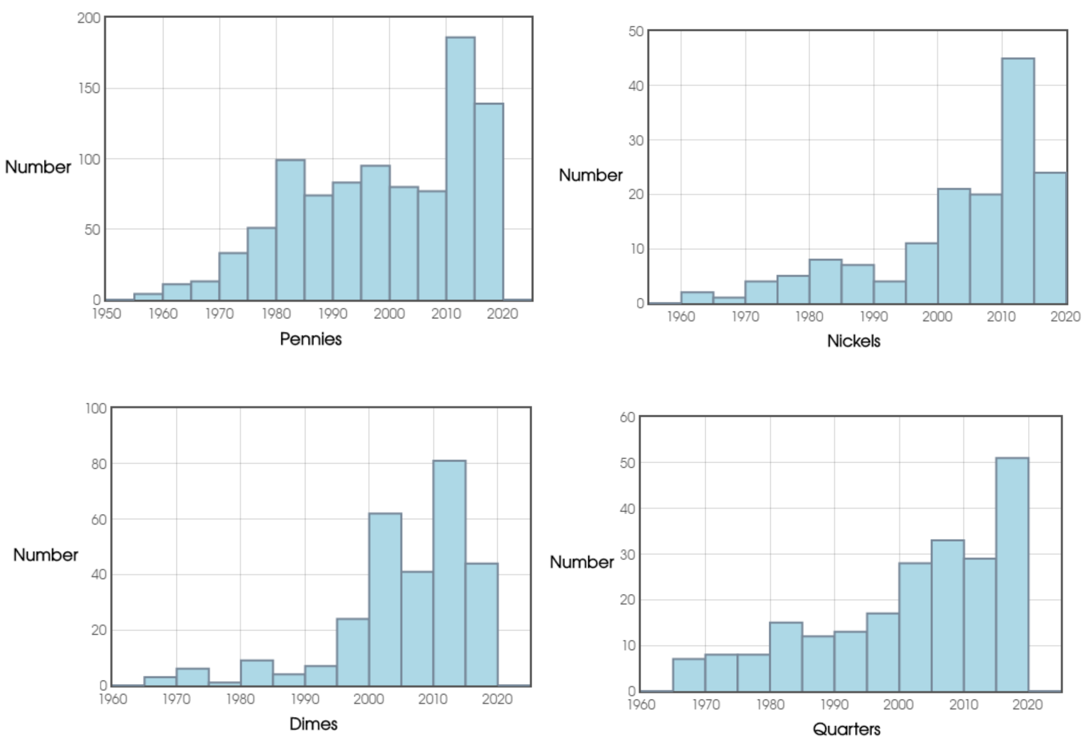
Median: M = 2005

3rd Quartile: Q3 = 2014

Maximum: Max = 2017



8. All of the distributions are very similar and obviously skewed to the left.



9. Total Coins: So we decided to also make a histogram and analyze the entire sample of coins

Number of Data Points: $n = 1600$

Mean: $\bar{x} = 2000.380625$

Standard Deviation: $S = 14.096273982758653$

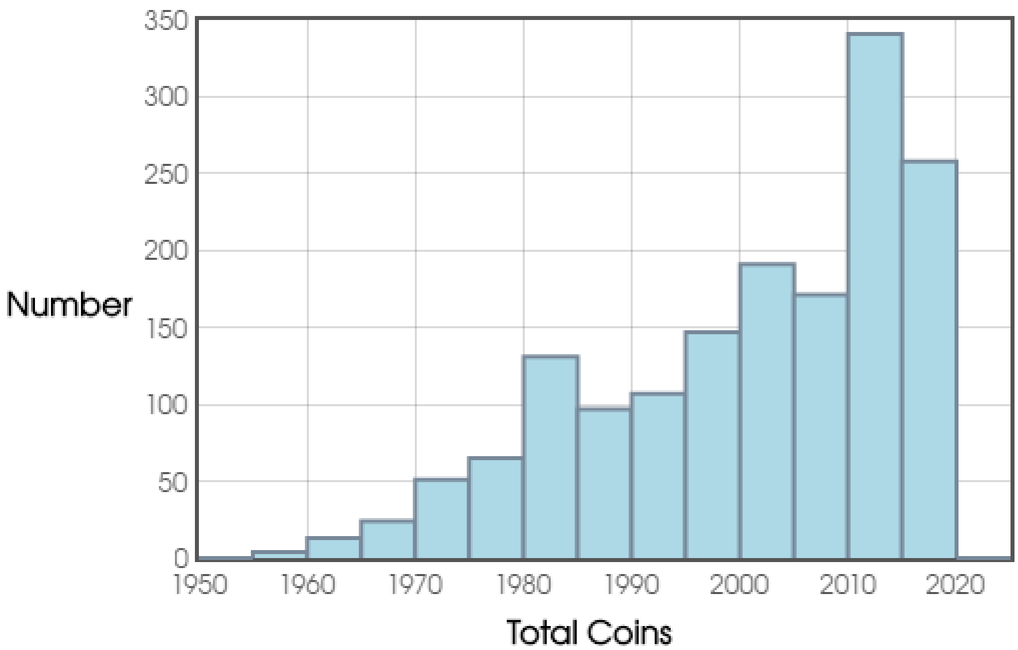
Minimum: Min = 1956

1st Quartile: Q1 = 1990

Median: M = 2004

3rd Quartile: Q3 = 2013

Maximum: Max = 2017



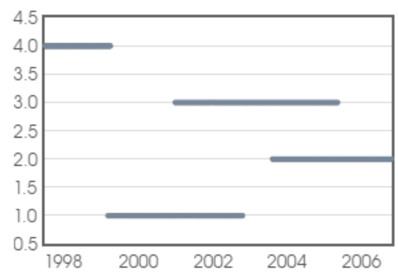
10. To compare all of the means of the different denominations of coins, we can use the Analysis of Variance (ANOVA) test. By inserting all of the data of the different denominations into the "**One-Way ANOVA**" calculator found at holt.blue, we can find the F-statistic and p-value.

Source of variation	df	SS	MS	F-statistic	p-value
Variation Among Samples	3	11690	MSG=3896.58	20.321	0
Variation Within Samples	1596	306039	MSE=191.75		
Total	1599	317729			

Sample	n	Mean	Standard Deviation	95% Conf. Interval using Pooled Std Dev
Pennies	945	1998.352	14.771	(1997.47, 1999.24)
Nickels	152	2003.171	13.081	(2000.97, 2005.37)
Dimes	282	2005.195	10.42	(2003.58, 2006.81)
Quarters	221	2000.991	14.1	(1999.16, 2002.82)

Pooled Std Dev: 13.848

95% Confidence Intervals Using Pooled Standard Deviation:



Results:

Using the data that we collected we were able to determine what the most common denomination of coin currency is in everyday circulation and what dates are most common in each denomination. Because of this experiment, we can be 95% certain that out of every 100 random coins selected out of our currency, 55-62 of them will be pennies, 4-14 of them will be nickels,13-22 of them will be dimes, and 9-18 of them will be quarters.

Within each denomination, we were able to determine the most common date and also if the mean date of each of the coin denominations varied from each other. With 95% certainty, we can be sure that the true mean date for pennies is between 1997 and 1999, the true mean date for nickels is between 2000 and 2005, the true mean date for dimes is between 2003 and 2006, and the true mean for quarters is between 1999 and 2002 (these were found using a pooled standard deviation).

As you can see, the mean dates of each denomination differ from one another. Using the ANOVA test, which measures the variance among and between denominations, we determined that these means had an F-statistic of 20.321. The P-value that corresponds with this number is basically 0. Because the F- statistic is so high and the P-value is so low, we can reject our null hypothesis which stated that all the means would be the same. This means that we can conclude that each denomination of coin has a different mean date.

<u>Explanation & Discussion:</u>

This could be because a larger amount of a certain coin was minted 5 years ago than it was 15 years ago, or visa-versa. Also, we must take into consideration that some coins have been removed from circulation from collectors and hoarders. This would make the less sought after coins more common in circulation. There are unlimited factors that could affect why we got the results that we did, too many to be listed. But the take-away is that a very specific amount of each coin gets made and added to the circulation every year and as coins get added to circulation other coins are getting removed. Therefore, the results of an experiment identical to this one, performed in 5 years, are going to have different results. Our currency has an ever-changing population. Every coin is different and has a unique history. By studying trends in our country's currency production, we can learn endless things. Coins can be like a newspaper, when studied correctly, they can tell you the economic, political, and military state of most countries.

References:

Haahr, Mads Dr., "List Randomizer." *RANDOM.ORG* , 25 March 2017, https://www.random.org/lists/.

Holt, Benjamin, "Statistical Software Suite.", *holt.blue*, 25 March 2017, holt.blue/Math_2/Resources/stats_suite.html.

Moore, David S., William Notz, and Michael A. Fligner. *The Basic Practice of Statistics*. New York: W.H. Freeman, 2015. Print.