

Finding Particular Solutions to $Ly = f(x)$

$f(x)$	Form of y_p
$a_n x^n + \cdots + a_1 x + a_0$	$x^\ell (A_n x^n + \cdots + A_1 x + A_0)$
$a e^{\alpha x}$	$x^\ell A e^{\alpha x}$
$(a_n x^n + \cdots + a_1 x + a_0) e^{\alpha x}$	$x^\ell (A_n x^n + \cdots + A_1 x + A_0) e^{\alpha x}$
$a \cos(\beta x) + b \sin(\beta x)$	$x^\ell (A \cos(\beta x) + B \sin(\beta x))$
$p(x) \cos(\beta x) + q(x) \sin(\beta x)$	$x^\ell (P(x) \cos(\beta x) + Q(x) \sin(\beta x))$
$a e^{\alpha x} \cos(\beta x) + b e^{\alpha x} \sin(\beta x)$	$x^\ell (A e^{\alpha x} \cos(\beta x) + B e^{\alpha x} \sin(\beta x))$
$p(x) e^{\alpha x} \cos(\beta x) + q(x) e^{\alpha x} \sin(\beta x)$	$x^\ell (P(x) e^{\alpha x} \cos(\beta x) + Q(x) e^{\alpha x} \sin(\beta x))$

where

$$\begin{aligned}
 p(x) &= a_n x^n + \cdots + a_1 x + a_0, & q(x) &= b_m x^m + \cdots + b_1 x + b_0 \\
 P(x) &= A_N x^N + \cdots + A_1 x + A_0, & Q(x) &= B_N x^N + \cdots + B_1 x + B_0, \quad N = \max\{n, m\}
 \end{aligned}$$

and ℓ is the smallest non-negative integer such that $y_p = x^\ell y_g$ has no terms which solve the homogeneous equation.