A Population Model and Temperature Model for XXXXXX, XX.

XXXXXXXX XXXX
Math 241
Month, Day, Year.

Introduction

This project concerns modeling both population and temperature data we were able to find at `wikipedia.org` about the author's hometown of XXXXXX, XX.

First, however, we shall tell the reader a little bit about XXXXXX. While it is not a large town, there is a lot history there. It started out as a mining camp by migrants seeking their fortune. They named the town XXXXXX since the area's terrain reminded them of their homeland [3].

It is a thoroughfare for many adventure seekers as it is a gateway to many sought-after destinations including XXXXXXXX National Park, XXXX Lake, and The XXXXXXX Wilderness to name a few. In the 20th century, timber took the place of mining as a principal economic driver of the region. However, since the 1990s, timber has been on the decline. Today, tourism fuels a large part of the economy of the region. Moreover, urban flight has caused the population to increase at historically unprecedented rates This was especially true in the 1980s, and the population continues to grow.

The above image is a recent photograph of XXXXXX, XX [3].

Now that the reader is more familiar with the author's hometown, we shall describe the main task of this project. We were asked to estimate the following:

1. The size of the population of XXXXXX, XX in 2020.
2. The rate at which the population is growing (or shrinking) in 2020.
3. The rate at which the average high temperature changes in May.
4. The rate at which the average high temperature changes in October.

Population

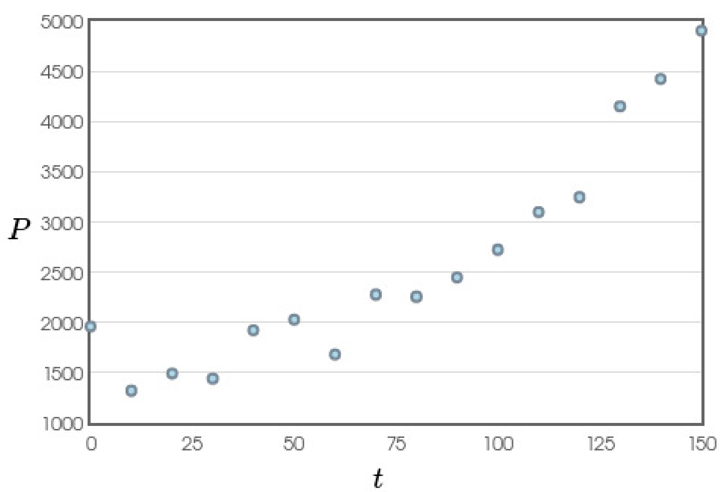
We shall first analyze the data we found in order to predict the size and rate of population growth of XXXXXX, XX. We used demographic data provided by Wikipedia [3], which is given below.

Historical population		
Census	Pop.	%±
1860	1,960	—
1870	1,322	−32.6%
1880	1,492	12.9%
1890	1,441	−3.4%
1900	1,922	33.4%
1910	2,029	5.6%
1920	1,684	−17.0%
1930	2,278	35.3%
1940	2,257	−0.9%
1950	2,448	8.5%
1960	2,725	11.3%
1970	3,100	13.8%
1980	3,247	4.7%
1990	4,153	27.9%
2000	4,423	6.5%
2010	4,610	4.2%
2019 (est.)	4,870 [4]	5.6%

U.S. Decennial Census[12]

We note that this same data is also available at `https://factfinder.census.gov`.

To analyze the above data, we used the website `http://holt.blue` [2], to generate our plots and to calculate the statistical models we will use to make our prediction for the population of XXXXXX, XX in 2020.

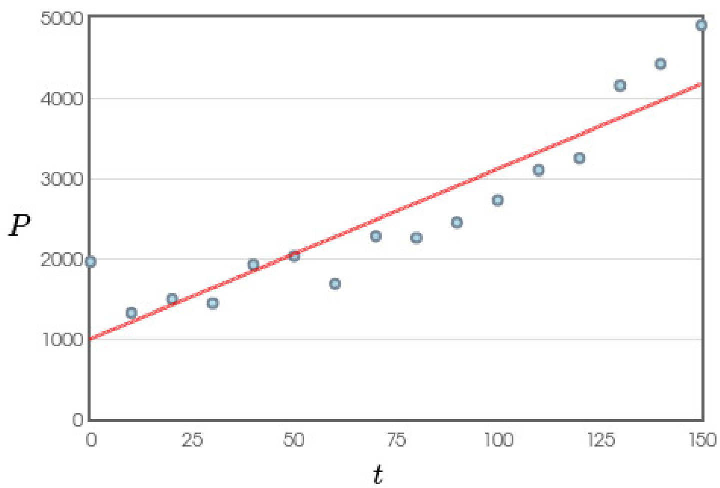


In the above figure, P denotes the number of people and t denotes number of years since 1860.

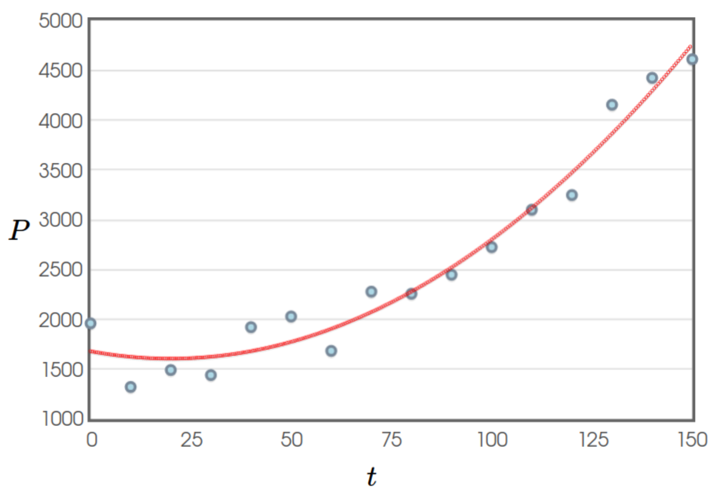
Since the data at first glance look like they could be linear, we first fitted a linear model,

$$P = 21.1735t + 998.4853,$$

which is plotted as the red line along with the population data.



Noticing that the model consistently underpredicts at the ends, and overpredicts in the middle, we decided to try a quadratic model instead. Running a least-squares regression on the data we obtained the following figure which contains both the data and the exponential curve of best fit in red.



The equation for the exponential curve of best fit to four decimal places is

$$P = 0.1864t^2 - 7.43t + 1680.9816.$$

Comparing both models visually, the reader can see that the quadratic model seems to be a better fit to the data than the linear model.

We note that we also tried fitting cubic and quartic (fourth degree) polynomial models, but none of these offered any extra predictive power, and the fit did not appear to be any better. Thus, we concluded that the quadratic model was the best model to use to make our prediction.

At this point, we may now answer the questions about the overall rate of growth of XXXXXX,XX and its predicted population for the year 2020:

In the year 2020, that is $t = 160$ for our model, we predict that XXXXXX, XX will have 5264 residents. We obtained this number by evaluating the model at $t = 160$: $P(160) = 0.1864(160)^2 - 7.43(160) + 1680.9816 \approx$ 5264 rounding to the nearest whole number.

We now estimate the rate at which the population of XXXXXX, XX will be growing in 2020. To find the rate of change of population with respect to time, we find the derivative of P with respect to t of our exponential model:

$$\frac{dP}{dt} = \frac{d}{dt}(0.1864t^2 - 7.43t + 1680.9816)$$

$$= 0.1864 \cdot 2t - 7.43$$

$$= 0.3728t - 7.43.$$

The last is an approximation to four decimal places. Evaluating the derivative function $P'(t) = 0.1864 \cdot 2t - 7.43$ at $t = 160$, we obtain $P'(160) \approx 52.218$ people per year.

We interpret the above to mean that in the year 2020, XXXXXX, XX will be growing at a rate of about 52 people per year.

Looking at the estimate for 2019, it appears that our numbers may be overpredictions. Considering the linear model again, it predicts a population of $P(160) = 21.1735(160) + 998.4853 \approx 4386$, rounding to the nearest whole number. Also, the rate at which the population is growing is 21 people per year. The numbers otained from our linear model are clearly an underpredictions keeping the 2019 estimation in mind.

From our linear and quadradic models, we estimate that the population of XXXXXX, XX in 2020 will be somwhere between 4386 and 5262 people and that it will be growing by about 21 to 52 people per year.

Given the lack of certainty of our estimate, we conclude that a better model is needed.

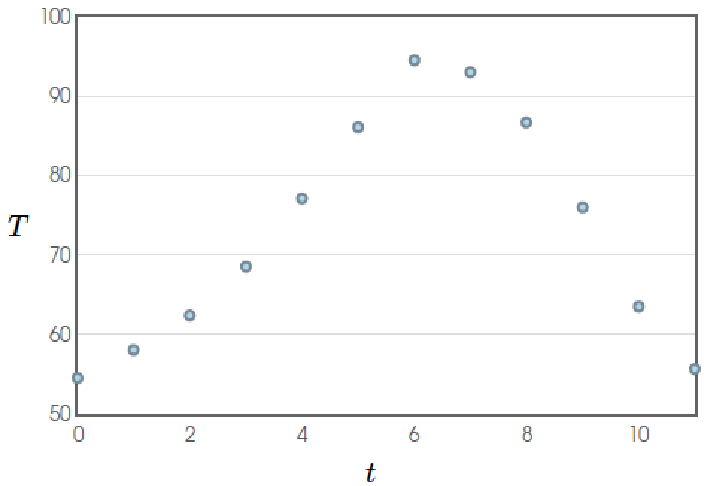
Temperature

We now move on to the average-high temperature of XXXXXX, XX. The data we used was again obtained from `wikipedia.org` [3], who in turn used data from The Weather Channel and The Desert Research Institute [4]. The data are displayed below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
Record high °F (°C)	75 (24)	78 (26)	84 (29)	92 (33)	103 (39)	113 (45)	113 (45)	110 (43)	108 (42)	100 (38)	89 (32)	81 (27)	113 (45)
Average high °F (°C)	54.5 (12.5)	58.0 (14.4)	62.4 (16.9)	68.5 (20.3)	77.1 (25.1)	86.1 (30.1)	94.5 (34.7)	93.0 (33.9)	86.7 (30.4)	76.0 (24.4)	63.5 (17.5)	55.6 (13.1)	73.0 (22.8)
Average low °F (°C)	33.4 (0.8)	35.5 (1.9)	38.0 (3.3)	41.7 (5.4)	46.7 (8.2)	52.7 (11.5)	58.7 (14.8)	57.4 (14.1)	52.7 (11.5)	45.2 (7.3)	38.1 (3.4)	33.8 (1.0)	44.5 (6.9)
Record low °F (°C)	13 (−11)	15 (−9)	20 (−7)	24 (−4)	24 (−4)	33 (1)	39 (4)	40 (4)	33 (1)	25 (−4)	21 (−6)	8 (−13)	8 (−13)
Average precipitation inches (mm)	6.13 (156)	5.55 (141)	5.10 (130)	2.77 (70)	1.27 (32)	0.33 (8.4)	0.04 (1.0)	0.08 (2.0)	0.39 (9.9)	1.62 (41)	3.46 (88)	5.39 (137)	32.14 (816)
Average snowfall inches (cm)	2.3 (5.8)	1.0 (2.5)	0.5 (1.3)	0.2 (0.51)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0.7 (1.8)	4.7 (12)

We will be modeling the average high temperature displayed in the second row of the above table.

Again, we analyzed our data using software available at `http://holt.blue` [2]. The scatter plot of the above data is given below.



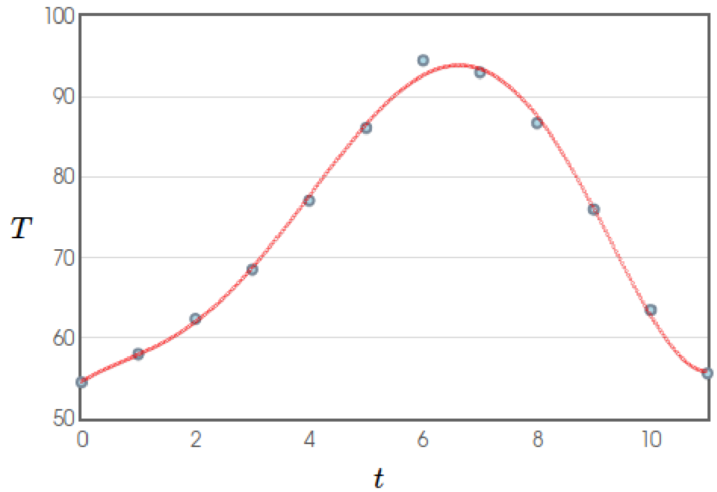
We note that temperature (in degrees Fahrenheit) is denoted T and time (in months) is denoted t.

Although the naturally periodic nature of the data lends itself naturally to using trigonometric functions, we obtained a very good fit using a polynomial model.

We now do a similar analysis using our temperature model. Performing a polynomial regression on our data, we obtained the following fifth-degree best-fit polynomial

$$T = 0.0078t^5 - 0.1843t^4 + 1.2416t^3 - 2.202t^2 + 4.4965t + 54.5822$$

which is graphed below in red along with the data.



Performing similar computations as above, we see that

$$\frac{dT}{dt} = \frac{d}{dt}(0.0078t^5 - 0.1843t^4 + 1.2416t^3 - 2.2020t^2 + 4.4965t + 54.5822)$$

$$= 0.0078 \cdot 5t^4 - 0.1843 \cdot 4t^3 + 1.2416 \cdot 3t^2 - 2.2020 \cdot 2t + 4.4965$$

$$= 0.039t^4 - 0.7372t^3 + 3.7248t^2 - 4.404t + 4.4965.$$

Now, $T'(4) \approx 9.28$. Thus, our polynomial model tells us that the average high temperature of XXXXXX increases by 9.28 degrees per month at the beginning of May.

Also, $T'(9) \approx -14.97$. Thus, our polynomial model estimates that the average high temperature is decreasing by 14.97 degrees per month at the beginning of October.

Summary and Conclusions

We have modeled both the population and average high temperature of XXXXXX, XX. We explored multiple models for each in order to find the model which best represents the data. Among the models available to us, we concluded that a quadratic model was the best for describing the population dynamics of XXXXXX, XX. We saw that by 2020, XXXXXX, XX From our linear and quadradic models, we estimate that the population of XXXXXX, XX will be somwhere between 4386 and 5262 people and that it will be growing by about 21 to 52 people per year. The wide predicitive range of these two models leads us to the conclusion that better models are needed to model the population of this town.

As for temperature, we constructed a fifth-degree polynomial which modeled the data well.

For the beginning of May, the polynomial model predicts 9.28 degrees per month.

For the beginning of October, the polynomial model predicts a rate of decrease of 14.97 degrees per month.

References

[1] Gilbert Strang, *Calculus: Volume 1*, Open Stax: `https://openstax.org/details/books/calculus-volume-1`, 2018.

[2] Holt.Blue Statistical Software Suite, available at http://holt.blue

[3] Wikpedia Data for XXXXXX, XX: https://en.wikipedia.org/wiki/XXXXXX,_XX

[4] Desert Research Institute Data, available at https://wrcc.dri.edu/cgi-bin/cliMAIN.pl?ca8353